Temperament Theory without Tears

Steve Brady

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When we began learning tuning, we learned by rote. Over the years Steve read more and learned more. Owen Jorgensen has written three books and there are others. Steve taught Music 301 for ten years for piano majors about how to tune a piano and tempering the scale. He would ask if they wanted to see the mathematics or hear a cruel joke of nature.

- **Cent:** One cent is 100th of a semi-tone in equal temperament, but it is actually 1/1200th of an octave.
- **Just:** The word "Just" means tune pure or beatless.
- **Perfect 5th**: an interval, as opposed to a diminished 5th or augmented 5th. Does not refer to how that interval is tuned.
- **Comma:** The difference between two sets or combinations of just intervals
- **Tempered**: altered from just tuning by a predetermined amount
- **Temperament**: one or more intentionally altered intervals in the tuning process.

Why do we need to temper the scale?

- Octave ratio: 2:1
- P5th ratio: 3:2
- C1 frequency x 2 (7 times)
- C1 frequency x 1.5 (12 times)

Tune by fifths and the octave above will be sharper than it would be when tuning by octaves.
B# becomes 24 cents sharper than C. Pythagoras discovered this dilemma.
To demonstrate, Steve tuned pure perfect fifths twelve times to compare how the octaves sound.
Diatonic means through the keys. The diatonic scale is how music evolved. Gradually people started adding black keys. A minor scale needs a raised seventh. Then in the parallel scale they needed another half tone. The year 1361 the keyboard became established with all the black keys.

The Commas and their Significance

- Ditonic Comma (Comma of Pythagoras): The excess of 12 just P5ths over 7 just octaves, approximately 24 c. Because of this comma, octaves and P6ths can't be in tune at the same time.
- Syntonic comma (Comma of Didymus): The excess of 4 just P5ths over two just octaves plus a just M3rd.

How are P5ths and M3rds Related?

- M2rds are generated by tuning a chain of 4 P5ths.
- C-G-D-A-E
- A chain of 4 Just P5ths results in a very wide M3rd (by a syntonic comma) called a Pythagorean 3rd (22 cents wide)
- A chain of 4 P5ths each tempered (narrowed) by 5.5 cents results in a pure, or just, M3rd.
- Steve demonstrated by tuning 4 P5ths tempered by 1/4 comma each. This produces a very quiet triad.
- The Major 3rds are what determine Key Color. Different keys were described with emotional feeling words.

In the Pythagorean Tuning, when people were primarily playing in C, F, C and G Major, the 24 cents off was hidden between G# and E flat. Major 3rds are very fast in order to enjoy pure 5ths. 1496 was the first mention of tempering.

The First Temperaments

- The first true temperament was documented by Grammateus in 1518. He split the comma between two 5ths. When a third crosses the half-comma (11c) it is slower. The tempered major thirds are only 11 cents (1/2 comma) wide.
- Regular temperament: all P5ths tempered the same amount
- Irregular temperament: different amounts
- Restricted system: one or more major or minor triads musically unusable

Historically there were two paths to equal temperament

- Meantone temperaments
  - Tempering by fractions of the Syntonic comma
  - The school of purity
  - True Meantone temperaments always contain a wolf (because it howls)
  - Playable in at least 2/3 of the major and minor keys
  - By subtracting ¼ comma between the first five 5ths, we end up with a really wide 5th, so they started tempering in the other direction. However, the wolf 5th ends up 1-3/4 cents out. In 1523, after Pythagoras.
  - Steve played the wolf fifth: actually a diminished 6th.
  - Steve played Bach's D-minor Toccata in Meantone temperament.
- Jean-Philippe Rameau temperament, 1726: a meantone derivative
  - Tempered by quarter commas, but he split the wolf on the back side, so there were two fifths one third of a comma wide, and a number of un-tempered fifths.
  - There are three different sizes of fifths. The worst third is 33 cents wide, but is still musically usable, compared to a regular meantone wolf.
- Well-Temperament
  - Narrow the fifths to get a good octave

    - Tempering by fractions of the Ditonic comma
    - The school of utility and key color
    - Nor wolf but plenty of distinct key colors
    - Playable in all major and minor keys
- Werckmeister III, 1691,
    - F-a and C-E are nearly pure.
    - He tempered only 4 5^{ths} by one quarter comma each
- Kimberger III, 1779
    - Pure major 3rd C-E
    - He moved one tempered 5th to another place
    - Pythagorean 3rds on back side, used for special effects for the more anguished key tones
- Vallotti, 1754
    - Tempered 6 5ths by 1/6 of a comma each
    - Used for a lot of Baroque
    - Some are purer than equal temperament,
    - Three Pythagorean 3rds
- Thomas Young #2, 1800
    - 1/6 comma temperament, similar to Vallotti
    - The last three are all pure, with no tempering
- Episode 1: Bradley Lehman (musicologist)
    - 1722 Bach squiggle , original alignment
    - The number of squiggles matched the 5ths. Each circle might signify different kinds of tempered 5ths
    - With this tuning there is only one really bad third, tempered by 1/12 cent.
- Unrestricted Regular Temperament by 1/12th Ditonic Comma (Equal Temperament)
    - All thirds are the same
    - All fifths are the same
    - Difficult to accomplish
    - This is the temperament we are all paid to do

Carl Radford, RPT, plays Bach B-flat minor prelude in the Thomas Young tuning.
Then he played the same piece in Equal Tempered tuning.
Classical vs. Modern Tunings
Classical well-tempered tuning: the more distant keys can sound emotional and painful
Equal Temperament: in equal those chords are more beautiful than painful.
We also listened to the Bach tuned with Lehman, Equal, and Werkmeister temperaments.